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for optimal asset allocation with value-at-risk
constraints**

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A single-period model and some empirical evidences for optimal asset allocation with value-at-risk constraints*

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ABSTRACT

In this paper, we consider the optimal asset allocation problems under VaR constraints. It is shown that the separation property holds to a certain extent. The optimal allocation of funds in risky assets is dependent on the distribution of the returns of risky assets and the VaR level, but independent of the acceptable loss ratio; the amount to be borrowed or lent at the risk free rate depends on the acceptable loss ratio. A general asset allocation model under VaR constraints is derived. As an application of our model, we address the optimal asset allocation between two categories of assets—bonds and stocks. Interesting empirical results are obtained for the US, Australia and the UK. The empirical results show that the mechanism of asset allocation under VaR constraints is fundamentally different from the classical mean-variance approach. The empirical results appear to support our model and demonstrate the potential usefulness of our approach.

Key words: Value at Risk, optimal asset allocation, separation property, empirical evidence.

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1. Introduction

How to measure risk is a critical issue in portfolio theory. The traditional approach is the mean-variance framework where risk is defined as the variation of a portfolio return. This approach has led to a number of advantages in mathematical manipulations and yielded very important results in optimal asset allocation. One important result is the so-called mutual funds theorem or the separation property (see e.g. Bodie et al, 1999). According to this theorem, all investors in an efficient market would invest in the same risky portfolio (the market portfolio), and the proportions invested in the risky portfolio and the risk free asset are determined by each investor's preference which can be illustrated by the utility function.

A major shortcoming in the traditional approach is that the risk measure used does not provide us enough information of the down-side risk which can be a major concern in risk management. To circumvent this problem, risk measures such as semi-variance were originally introduced. Another approach that has been taken is to incorporate risk directly into the asset allocation model. The optimal portfolio is then selected by maximising the expected return over candidate portfolios so that some shortfall criterion is satisfied. Leibowitz and Kogelman (1991) and Lucas and Klassen (1998) construct portfolios by maximising return subject to a shortfall constraint, which is defined such that a minimum return should be gained over a given time horizon for a given confidence level. Roy (1952) and Arzac and Bawa (1977) define the shortfall constraint such that the probability that the value of the portfolio falling below a specified disaster level is limited to a specified disaster probability. These are examples of the safety-first approach to asset allocation. Although the concept of using shortfall constraints is more in line with investors' perception to risk, their applicability is rather limited since the disaster levels, minimum returns, confidence levels or disaster probabilities are hard to specify.

Value at Risk (VaR) has become a key tool for risk management of financial institutions (Jorion, 2001). VaR describes the loss that can occur over a given period, at a given confidence level, due to exposure to market risk. The regulatory environment and the need for controlling risk in the financial community have provided incentives for banks to develop proprietary risk measurement models. Among other advantages, VaR provides quantitative and synthetic measures of risk, that allow to take into account various kinds of cross-dependence between asset returns, fat-tail and non-normality effects, arising from the presence of financial options or default risk etc.

The wide usage of the VaR-based risk management by financial as well as non-financial firms stems from the fact that VaR is an easily interpretable summary measure of risk and also has an appealing rationale, as it allows its users to focus attention on "normal market conditions" in their routine operations. Due to the wide application of VaR, the research on VaR has been very active in recent years, for a summary we refer to Duffie and Pan (1997).

In this paper, we are concerned with asset allocation problems subject to VaR constraints. Some attempts have been made in tackling these optimization problems under various assumptions in recent years. For example, Basak and Shapiro (2001) analyzes optimal, dynamic portfolio and wealth/consumption policies of utility maximizing investors who also must manage market-risk exposure using the VaR. They focus on portfolio choice within the continuous time complete market setting. In contrast, we concentrate on a simpler but nevertheless useful situation as considered by Huisman et al (1999). We consider a single period model where investors' objective is to maximise their expected wealth at the end of the period, under certain VaR constraints.

Throughout this paper, we assume that investors can borrow or lend at the risk free rate and there is no restriction on the amount that can be borrowed or lent. We also assume that short sale of the risky asset(s) is allowed.

We first consider a model where there is only one risky asset. In this case, we need only to find the leverage ratio. Given an acceptable VaR, the optimal leverage ratio can be given explicitly and the amount invested in the risky asset can be determined accordingly. We then move to a more complicated situation where there are multiple risky assets. In this case, we need to deal with two decisions. One is to determine the leverage ratio which indicates how much should be borrowed or lent as a percentage of an investor's initial wealth. Another decision is how to allocate the fund among the risky assets. It turns out that a similar result as the mutual funds theorem holds for the above investment decision under VaR constraint. We show that the optimal investment proportions in risky assets are independent of the acceptable loss ratio, but dependent on the VaR level. The optimal solution is found by maximising an index which is similar to the Sharpe ratio in the classical mean-variance approach. The leverage ratio is then determined by the acceptable loss ratio and the index value. Thus our results can be viewed as an extension of the previous asset allocation results under the mean-variance approach.

In Section 4, we show some applications of our model. One basic problem faced by funds managers is to determine the optimal asset allocation between bonds and equities as well as the leverage ratio for the investments under certain VaR constraints. We treat this problem as a two risky asset problem by using appropriate bond index and share index. We use the historical data for the US, Australia, and the UK and obtain the optimal allocation parameters (leverage ratios and investment proportions) for these countries. Our results are also contrasted with the asset allocations under the mean-variance approach. In section 5, we summarise the main conclusions.

The key contributions of this paper are in the following aspects. We undertake a comprehensive analysis of the single-period optimal asset allocation problems with VaR constraints and show that the separation property holds to a certain degree. Although our model is similar to Huisman et al. (1999), our approach is somehow different and our result is presented in terms of the acceptable loss ratio and the leverage ratio. A neat relation is also discovered for these ratios. Secondly, we obtained interesting empirical results which

demonstrate the complex nature of the asset allocation problem under VaR constraints. The empirical results are analyzed and found to be strongly supportive to our theoretical results.

2. A single risky asset model

In this section, we consider a single risky asset model. We assume that an investor has an initial wealth of W_0 and requires a VaR of V^* at level α , i.e., the maximum acceptable loss should not exceed V^* with a confidence level of $1-\alpha$. Furthermore, we assume that the market fulfils the following assumptions:

- i) The investor can borrow or lend any amount at the risk free rate r_f .
- ii) There is only one risky asset whose return is denoted by r_s .
- iii) There are no transaction costs and tax etc.

We assume that the investor wish to maximise his expected wealth subject to the constraint $VaR(\alpha) < V^*$, where $VaR(\alpha)$ denote the value at risk at the level α . Further, we assume that the investor may borrow money at the risk free rate and invest the proceeds together with his initial wealth in the risky portfolio, or the investor may split his initial wealth between the risk free asset and the risky asset. The problem thus reduces to the following question: how much should the investor borrow from or lend to the cash market?

To address this question, let us first introduce some notations. The return of the risky asset is a random variable r_s , whose distribution is known. In order to guarantee that the above optimal investment problem makes sense, we need to assume that $E(r_s) > r_f$, where $E(\cdot)$ is the expectation operator. We also need to assume that there is a significant chance that the risky asset will end up with a loss, i.e. $P(r_s < 0) > 0$, where $P(\cdot)$ denotes the probability for an event.

Let B denote the amount the investor borrows (if $B > 0$) or the amount the investor lends (if $B < 0$). Then the total amount the investor invests in the risky asset is: $W_0 + B$.

Note that VaR is defined as the worst expected loss over the chosen time horizon within a given confidence level (see Jorion, 2001). For example, a 1% VaR for a 10-day holding period, implies that the maximum loss incurred over the next 10 days should only exceed the VaR limit once in every 100 cases. Thus the constraint for the investor can be formulated by the following formula:

$$P(W_0 - W_1 \geq V^*) \leq \alpha \quad (1)$$

Assume that the investor borrows the amount B and invests this amount and his initial wealth in the risky asset for a specific period. Then at the period end, the wealth of the investor will be:

$$W_1 = (W_0 + B)(1 + r_s) - B(1 + r_f) = W_0(1 + r_s) + B(r_s - r_f). \quad (2)$$

Using this expression, the asset allocation problem can be formulated as an optimisation problem:

$$\max_B E(W_1) = E((W_0 + B)(1 + r_s) - B(1 + r_f))$$

Subject to $\text{VaR}(\alpha) < V^*$

By the definition of VaR, we have

$$\begin{aligned} P(W_0 - W_1 \geq V^*) &= P(-W_0 r_s - B(r_s - r_f) \geq V^*) \\ &= P(r_s \leq r_f - \frac{V^* + W_0 r_f}{B + W_0}) \leq \alpha. \end{aligned} \quad (3)$$

Let Φ be the cumulative distribution function of the return r_s of the risky asset. Define $q(\alpha) = \Phi^l(\alpha)$.

Note $E(r_s) > r_f$. The parameter B that maximizes $E(W_1)$ is given by:

$$\begin{aligned} B &= -W_0 + \frac{V^* + W_0 r_f}{r_f - q(\alpha)} \\ &= \frac{V^* + q(\alpha)W_0}{r_f - q(\alpha)} = \frac{\theta + q(\alpha)}{r_f - q(\alpha)} W_0 \end{aligned} \quad (4)$$

where θ is the acceptable loss ratio defined as:

$$\theta = \frac{V^*}{W_0}. \quad (5)$$

Further, we introduce the leverage ratio:

$$l = \frac{B}{W_0} \quad (6)$$

Then (4) is equivalent to

$$l = -1 + \frac{\theta + r_f}{r_f - q(\alpha)} = \frac{\theta + q(\alpha)}{r_f - q(\alpha)}. \quad (7)$$

This formula relates the acceptable loss ratio (θ) with the leverage ratio l . As the definition for B , a negative value of l implies lending while a positive one stands for borrowing. Note that the initial wealth W_0 and VaR loss V^* do not appear explicitly in Equation (7). Instead, they are replaced by the dimensionless parameters θ and l .

In this single risky asset model, the solution is given explicitly by Equation (7) for a given distribution of the return of the risky asset. It should be noted that we do not need any assumption on the distribution of the return of the risky asset in the above derivation. In the next section, we shall consider the multiple risky assets model by using the results we obtained above.

3. A multiple risky assets model

Now we consider the multiple risky assets model. We assume that there are n risky assets with returns denoted by r_1, r_2, \dots, r_n . Let the investment proportion in these assets be x_1, x_2, \dots, x_n , respectively. Then the risky portfolio return r_p can be represented as

$$r_p = \sum_{i=1}^n x_i r_i \quad (8)$$

The problem is now more complicated. We need to determine both the investment proportions and the amount to be lent or borrowed. However, as the mutual funds theorem in the classical investment theory, the solution for the multiple risky assets problem can be separated into two steps. If the optimal risky portfolio is determined, then we can treat the risky portfolio as the only risky asset in the above model and the problem can actually be reduced to the single risky asset problem for which we already have a solution given by Equation (7). Thus we need only to find the optimal risky portfolio.

As before, we let B be the amount to be borrowed or lent, Φ be the cumulative distribution function of r_p and $q(\alpha, r_p)$ be the inverse of the cumulative distribution function of the risky portfolio at α , i.e.

$$q(\alpha, r_p) = \Phi^{-1}(\alpha).$$

Then the borrowing amount will be given by the same formula as before, for a given risky portfolio. However, we now need to determine the parameters x_1, x_2, \dots, x_n . These proportions in turn determine the distribution of the risky portfolio. Similar to (4), we have

$$\begin{aligned} B &= -W_0 + \frac{V^* + W_0 r_f}{r_f - q(\alpha, r_p)} \\ &= \frac{V^* + q(\alpha, r_p) W_0}{r_f - q(\alpha, r_p)}. \end{aligned} \quad (9)$$

Thus the wealth at the period end is:

$$\begin{aligned} W_1 &= (1 + r_p)(W_0 + B) - B(1 + r_f) \\ &= W_0(1 + r_f) + (W_0 + B)(r_p - r_f) \\ &= W_0(1 + r_f) + \frac{V^* + W_0 r_f}{r_f - q(\alpha, r_p)}(r_p - r_f). \end{aligned} \quad (10)$$

Note that $q(\alpha, r_p)$ is only dependent on the numbers x_1, x_2, \dots, x_n and the distributions of r_1, r_2, \dots, r_n . Thus it is not a random number. Therefore we have:

$$\begin{aligned} E(W_1) &= W_0(1 + r_f) + E\left[\frac{V^* + W_0 r_f}{r_f - q(\alpha, r_p)}(r_p - r_f)\right] \\ &= W_0(1 + r_f) + \frac{V^* + W_0 r_f}{r_f - q(\alpha, r_p)}(E(r_p) - r_f). \end{aligned} \quad (11)$$

This implies that maximising the expected wealth at the end of the period is equivalent to maximising the index

$$S(x_1, x_2, \dots, x_n) = \frac{E(r_p) - r_f}{r_f - q(\alpha, r_p)} = \frac{\sum_{i=1}^n x_i E(r_i) - r_f}{r_f - q(\alpha, r_p)}. \quad (12)$$

Thus the optimal asset allocation problem can be formulated as:

$$\underset{(x_1, x_2, \dots, x_n)}{\text{Max}} \quad S(x_1, x_2, \dots, x_n) = \frac{\sum_{i=1}^n x_i E(r_i) - r_f}{r_f - q(\alpha, r_p)},$$

$$\text{Subject to} \quad \sum_{i=1}^n x_i = 1.$$

Note that the investment proportions in the risky assets are independent of the initial wealth, it depends only on the required level of VaR, risk free rate of return, the returns of the risky asset. This is quite similar to the well-known separation property in investments under the traditional mean-variance approach. After the investment proportions are obtained, the borrowing or lending amount can be obtained from (9). Combining (9) and (12) yields

$$l = -1 + \frac{S * (\theta + r_f)}{E(r_p) - r_f} \quad (13)$$

where l is the leverage ratio and θ is the acceptable loss ratio as defined by (5) and (6), respectively. Note that (13) gives l in terms of θ and S .

So far we have established a complete solution procedure to the optimal asset allocation problem with VaR constraints. It is interesting to compare the above model with the traditional mean-variance approach. Note that the mutual funds theorem or the separation property in the mean-variance approach states that

- i) For a given risk free rate, all investors invest in the same risky portfolio, i.e. the market portfolio.
- ii) The invest proportions in risk free asset and risky portfolio are determined by each individual's utility indifference function.

Our model is slightly different. The risky portfolio is dependent on an investor's VaR level α and the distribution of the return of the risky assets. But it is independent on the acceptable loss ratio θ . Further, the leverage ratio is dependent on the acceptable loss ratio and the index S . In sum, investments with VaR constraints in our setting are somewhat "semi-separable".

4. Empirical results on optimal asset allocations

In this section we provide empirical examples in which a portfolio manager needs to allocate funds to stocks and bonds such that certain VaR constraints are met. We shall consider three countries: the US, Australia and the UK. We shall contrast our results with that of the traditional mean-variance approach. To this end, we first recall the following results from Bodie et al (1999).

4.1 Optimal portfolio with two risky assets: maximising the Sharpe's ratio

Suppose that there are only two assets available for investments and one risk free asset. Let r_1, r_2 and r_f be the returns of the risky assets, the risk-free asset respectively. Further setting

$$R_1 = E(r_1) - r_f$$

$$R_2 = E(r_2) - r_f$$

Then the optimal asset allocation that maximises the Sharpe Ratio (cf Sharpe, 1994) is given by

$$x_1 = \frac{R_1 \sigma_2^2 - R_2 \rho \sigma_1 \sigma_2}{R_1 \sigma_2^2 + R_2 \sigma_1^2 - (R_1 + R_2) \rho \sigma_1 \sigma_2}, \quad (14)$$

$$x_2 = \frac{R_2 \sigma_1^2 - R_1 \rho \sigma_1 \sigma_2}{R_1 \sigma_2^2 + R_2 \sigma_1^2 - (R_1 + R_2) \rho \sigma_1 \sigma_2} \quad (15)$$

where σ_1, σ_2 are the standard deviation of r_1, r_2 , respectively, ρ is the correlation coefficient between the returns of the two risky assets. (14) and (15) can be obtained easily by using some basic calculus. We ignore the details of the derivation here.

Note that if we take one of the two assets as the risk-free asset in (14) and (15), then both the numerators and denominators of (14) and (15) become 0. Thus both (14) and (15) do not make sense in this special case. The above holds only for two risky assets. However, for the case of one risk free asset with a risky asset (i.e., the single risky asset model), the investment proportion can be derived similarly as above. It can be shown that in this case one need to invest 100% in the risky asset in order to maximise the Sharpe ratio.

It should also be noted that under the classical mean-variance approach, all investors will hold the same risky portfolio (the so called market portfolio) which is given by (14) and (15). However, the optimal amount of wealth invested in the optimal risky portfolio is determined by each investor's utility indifference curve. For details, see e.g. Bodie et al (1999).

4.2 Data and descriptive statistics

In order to illustrate the application of our model, we consider the case of two risky assets which are represented by bonds and stocks. This is a typical problem faced by funds managers who need to split funds into two categories: bonds and stocks. A funds manger often invests in a large number of bonds and stocks. Therefore we can use a bond index and a stock index to represent these two categories. In this paper we consider such asset

allocation problems in three countries: the US, Australia and the UK. All our data are taken from Datastream.

For the US, we use the S&P 500 Composite Return Index and the 10-year Datastream Benchmark US Government Bond Return Index. The daily data for a period of 10 years from July 1991 to July 2001 are taken. Thus we have a total of 2611 observations.

For Australia, we take the ASX All Ordinaries Index and the Australia Benchmark 10 year Datastream Government Return Index. However, the bond index is not available before June 1992. Thus we take the daily data for the period from June 1992 to July 2001, which gives a total of 2381 observations.

In the case of UK, we take the FTSE 100 and the UK Benchmark 10 Year Datastream Government Index. Due to the availability of the bond index, we use the daily data from January 1992 to July 2001, which provides a total of 2476 observations.

In order to understand our empirical results, it is necessary to first understand the key characteristics of the data to be used. A summary statistics for the returns of bonds and stock indices are given in Table 1. For each country, the average bond index return is less than the average stock index return, but bond index also has a higher standard deviation. That is, neither the bond index nor the stock index dominates the other in the mean-variance framework for each country. The skewness figures show that the returns of bond index and stock index for all countries are skewed to the upper tail. According to the kurtosis figures in Table 1, the return distributions for both bond index and stock index in each country are more peaked relative to the normal distribution. It appears that bond index returns are more peaked than stock index returns for all three countries.

The distribution profiles of the bond and stock indexes for US, Australia and UK are plotted in Figures 1, 2 and 3 respectively. The skewness and peakness of these profiles can be easily observed from Figures 1-3. In other words, all return distributions are very different from the normal distribution as often assumed. This confirms that it is more reasonable to use the historical returns for these distributions instead of the normal distribution assumption.

A careful comparison of Figures 1, 2 and 3 reveals that the distribution profiles of bond index and stock index returns for all countries are quite similar. However, there are delicate differences in the distribution profiles. These differences may account for some differences of optimal asset allocation in the three countries.

4.3 The empirical results

Applying our model to the above historical index data, we obtain the optimal asset allocation parameters for the US, Australia and the UK. The results are summarised in Tables 2-5. For these three countries, various interest rates (the risk free rates) ranging from 3% p.a. to 7% p.a. are considered. Optimal asset allocations are obtained for VaR level of 1%, 5% and 10%. Then the leverage ratios are obtained by assuming a 5% acceptable loss

ratio. Besides, the optimal asset allocations under the mean-variance approach are also calculated for contrast.

Now let us analyse the results in more details. First, consider the US market. On the 1% VaR level, the optimal investment proportion in bonds is 64% to 68% for interest rates ranging from 3% and 4%. However, the investment proportion in bonds is stable at 44% and leverage ratio decreases for interest rate ranging from 4% to 6.5%. The investment proportion in bond dropped sharply to 20% when the interest rate increases to 7%. The varying trend of investment proportion in bonds is consistent with the common sense: bonds are normally less risky than stocks and bonds are becoming less attractive as the interest rate goes up. For a given acceptable loss ratio of 5%, the leverage ratios are monotonic and range from 1.47 to 3.54 for various interest rates from 3% to 7%. This is in accordance with the fact that borrowing is more desirable at a lower interest rate. Similar trends hold for 5% and 10% VaR levels.

Figure 4 shows that the dependence of the optimal asset allocation on the VaR level α is very complicated. It does not show any monotonicity in α . This is largely due to the changes in the leverage ratios for different α 's. Similar results are obtained for Australia and UK, see Figures 5 and 6. The complexity can also be observed from the nonlinear feature of Equation (12).

In all three cases of VaR levels (1%, 5% and 10%), the resulting asset allocations are very different from the results implied by the mean-variance approach for the three countries considered, see Figures 4, 5 and 6. This fact shows that the investment allocation mechanism under VaR constraints is fundamentally different from the conventional mean-variance approach.

The Australia empirical results reveal similar trends as the US data, see Table 3 and Figure 5. However, the asset allocation for Australian market seems quite insensitive to interest rates in the range from 3% to 7%. For example, at the VaR level of 1%, the optimal asset allocation for Australia market turns out to be a constant 48% in bonds and a constant 52% in equities. For a given acceptable loss ratio of 5%, the leverage ratio in this case is also nearly a constant, the numbers range from 2.53 to 2.58.

For the UK market, the same trends hold, see Table 4 and Figure 6. The allocation proportions in the UK seem to be less stable than Australia, but more stable than the US.

In sum, our empirical results for the US, Australia, and the UK reveal the following general observations regarding the optimal asset allocation problem under VaR constraints.

- i) All else being the same, the allocation proportion in bonds should increase, and the allocation proportion in stocks should decrease as the interest rate increases.
- ii) All else being the same, the leverage ratio for optimal investment under VaR constraint decreases as interest rate increases.

- iii) The dependence of optimal asset allocation on the VaR level α is very complex and highly nonlinear in nature.
- iv) The mechanism of asset allocation under VaR constraints is fundamentally different from asset allocation under the traditional mean-variance approach.

5. Conclusions

Since the introduction of VaR as a risk management tool recommended by the Basle committee, the VaR concept has gained great popularity. For example, it has become an industry standard for banks and many financial institutions to report their VaR regularly. As such, the asset allocation problem under VaR constraints is becoming an important issue. As a risk measure, the key difference between VaR and standard deviation is that VaR measures explicitly the down-side risk.

Modern portfolio theory has established various results with standard deviation of assets returns as a measure of risk. However, there are not many results available yet in the VaR framework. In this paper, we provide a model for asset allocation which is able to move away from the use of standard deviation alone as the appropriate measure for risk in financial markets. We focus on the use of down-side risk, as measured by VaR and hence we are able to allocate assets more in accordance with the risk perceptions which investors hold. The model is derived without having to impose distributional assumptions about the future distribution of returns. We show that the separation property which holds in the mean –variance approach is still valid to a certain degree.

Empirical results for the US, Australia and the UK have provided the evidence of additional down-side risk from skewness and kurtosis, thus the use of the normal distribution results in an incorrect estimation of the risk-return trade off for investors wanting to know the probability of their portfolio falling below the VaR level with high confidence.

Finally our empirical results also reveal that the dependence of the investment proportions on the VaR level has a complex nature, which calls for further research.

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Table 1. Summary statistics of bond and stock index daily returns

	US		Australia		UK	
	Bonds	Stocks	Bonds	Stocks	Bonds	Stocks
Mean	0.0278%	0.0521%	0.0362%	0.0424%	0.0359%	0.0444%
Median	0.0200%	0.0215%	0.0254%	0.0258%	0.0263%	0.0185%
Maximum	1.6659%	4.9948%	2.6034%	5.9197%	3.5799%	5.4584%
Minimum	-2.8335%	-7.1066%	-3.0371%	-6.9862%	-2.2684%	-4.1651%
St. Deviation	0.4059%	0.9574%	0.5429%	0.8252%	0.4255%	0.9573%
Skewness	-0.4242	-0.2796	-0.3927	-0.3592	-0.0347	-0.0500
Kurtosis	2.7603	5.3402	2.6997	4.7320	4.4876	2.0712
Sample size	2610	2610	2380	2380	2475	2475

Figure 1. The distribution of US bond and stock index returns

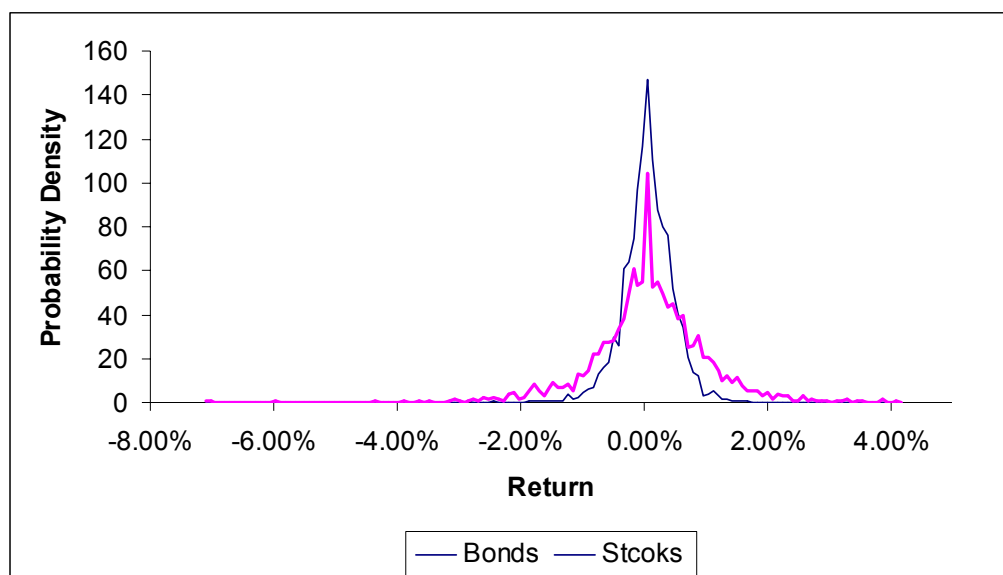


Figure 2. The distribution of Australia bond and stock index returns

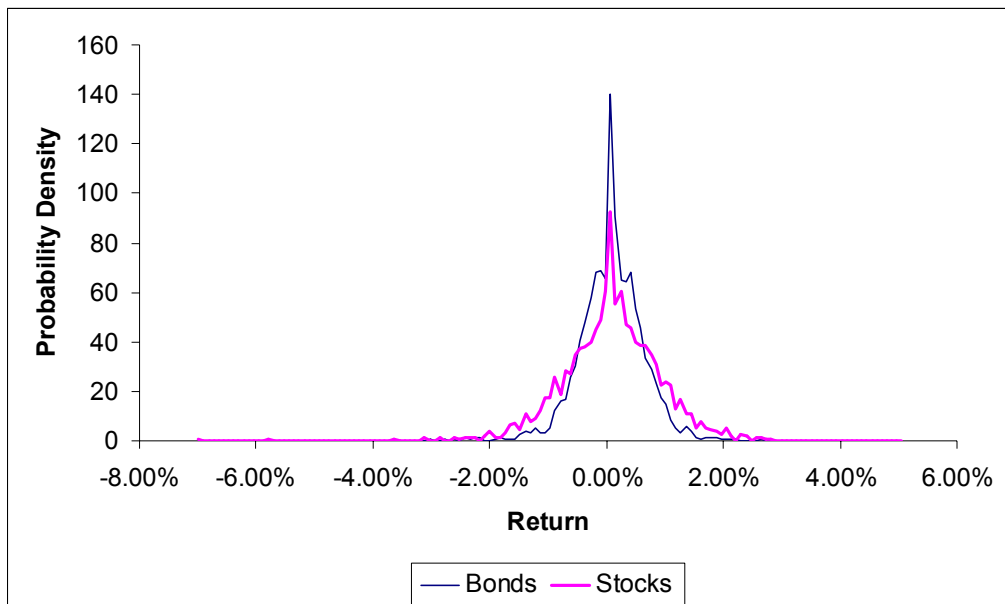


Figure 3. The distribution of UK bond and stock index returns

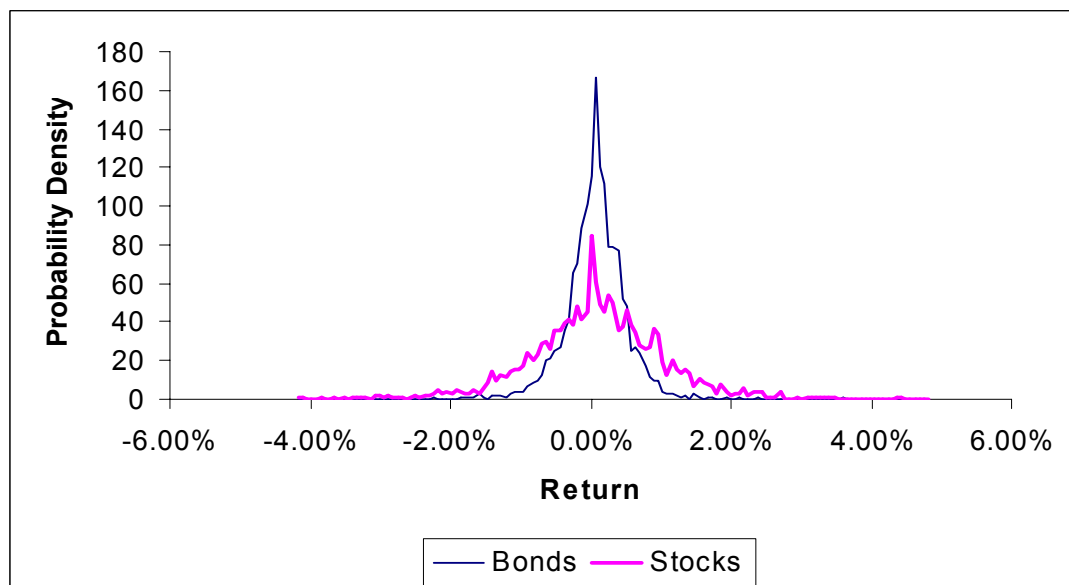


Table 2. Optimal asset allocation in the US

Interest Rate	3%	3.5%	4%	4.5%	5%	5.5%	6%	6.5%	7%
VaR Model, $\alpha=1\%$, $\theta=5\%$									
Bonds	68%	68%	64%	44%	44%	44%	44%	44%	20%
Stocks	32%	32%	36%	56%	56%	56%	56%	56%	80%
Leverage ratio	3.54	3.53	3.33	2.45	2.44	2.44	2.44	2.43	1.47
VaR Model, $\alpha=5\%$, $\theta=5\%$									
Bonds	73%	73%	62%	62%	62%	36%	34%	-1%	-7%
Stocks	27%	27%	38%	38%	38%	64%	66%	101%	107%
Leverage ratio	6.78	6.76	5.82	5.81	5.79	3.85	3.73	2.24	2.21
VaR Model, $\alpha=10\%$, $\theta=5\%$									
Bonds	71%	71%	71%	28%	10%	4%	4%	-14%	-32%
Stocks	29%	29%	29%	72%	90%	96%	96%	114%	132%
Leverage ratio	9.57	9.53	9.49	5.58	4.61	4.32	4.31	3.55	2.96
Mean-Variance Model									
Bonds	69%	66%	63%	59%	54%	46%	33%	10%	-47%
Stocks	31%	34%	37%	41%	46%	33%	67%	90%	147%

Figure 4. Optimal asset allocation in the US

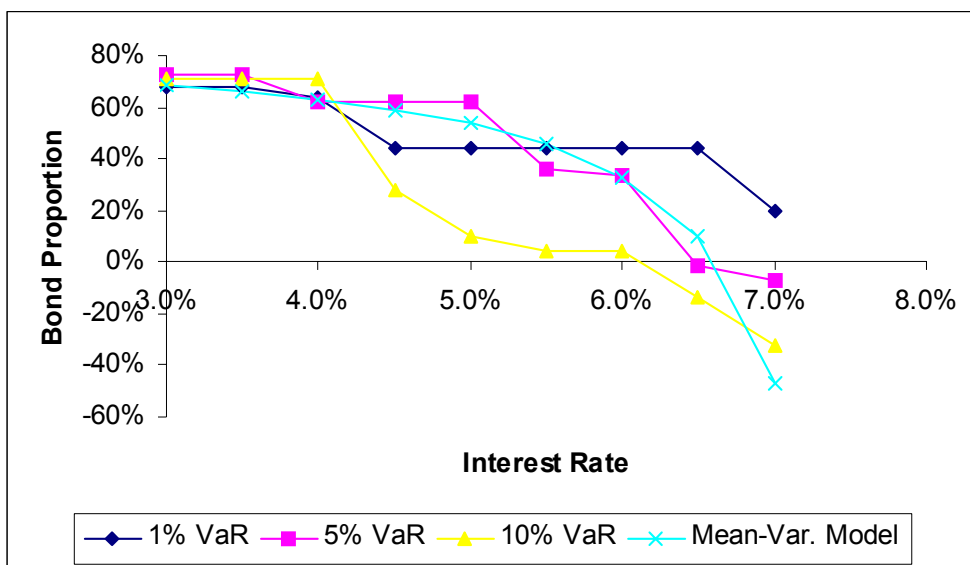


Table 3. Optimal asset allocation in Australia

Interest Rate	3%	3.5%	4%	4.5%	5%	5.5%	6%	6.5%	7%
VaR Model, $\alpha=1\%$, $\theta=5\%$									
Bonds	48%	48%	48%	48%	48%	48%	48%	48%	48%
Stocks	52%	52%	52%	52%	52%	52%	52%	52%	52%
Leverage ratio	2.58	2.55	2.55	2.55	2.54	2.54	2.54	2.53	2.53
VaR Model, $\alpha=5\%$, $\theta=5\%$									
Bonds	72%	72%	72%	72%	72%	72%	54%	54%	54%
Stocks	28%	28%	28%	28%	28%	28%	46%	46%	46%
Leverage ratio	5.52	5.51	5.49	5.48	5.46	5.45	5.00	4.99	4.98
VaR Model, $\alpha=10\%$, $\theta=5\%$									
Bonds	79%	79%	79%	65%	65%	62%	62%	62%	62%
Stocks	21%	21%	21%	35%	35%	38%	38%	38%	38%
Leverage ratio	8.30	8.27	8.24	7.83	7.80	7.68	7.63	4.31	7.60
Mean-Variance Model									
Bonds	68%	67%	66%	65%	64%	63%	61%	58%	55%
Stocks	32%	33%	34%	35%	36%	37%	39%	42%	45%

Figure 5. Optimal asset allocation in Australia

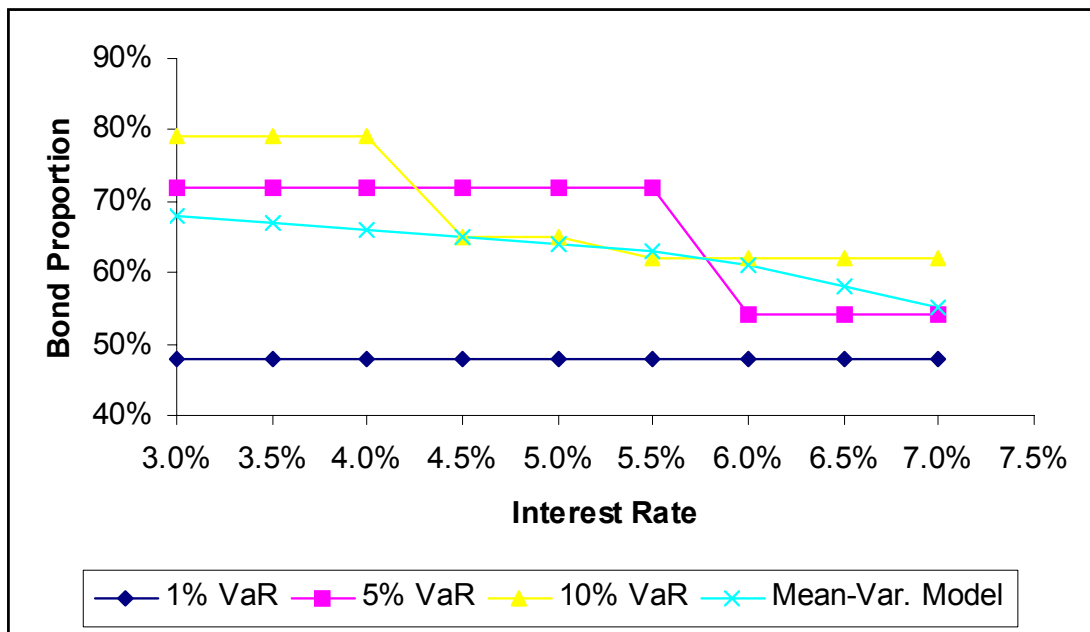


Table 4. Optimal asset allocation in UK

Interest Rate	3%	3.5%	4%	4.5%	5%	5.5%	6%	6.5%	7%
VaR Model, $\alpha=1\%$, $\theta=5\%$									
Bonds	74%	74%	74%	74%	74%	59%	55%	55%	48%
Stocks	26%	26%	26%	26%	26%	41%	45%	45%	52%
Leverage ratio	2.91	2.91	2.91	2.90	2.90	2.63	2.56	2.55	2.38
VaR Model, $\alpha=5\%$, $\theta=5\%$									
Bonds	84%	84%	84%	84%	83%	83%	83%	68%	68%
Stocks	16%	16%	16%	16%	17%	17%	17%	32%	32%
Leverage ratio	6.62	6.60	6.58	6.57	6.50	6.48	6.46	5.77	5.75
VaR Model, $\alpha=10\%$, $\theta=5\%$									
Bonds	88%	88%	88%	88%	88%	88%	80%	80%	80%
Stocks	12%	12%	12%	12%	12%	12%	20%	20%	20%
Leverage ratio	10.26	10.21	10.16	10.12	10.07	10.03	9.48	9.44	9.40
Mean-Variance Model									
Bonds	84%	84%	83%	82%	81%	80%	78%	76%	72%
Stocks	16%	16%	17%	18%	19%	20%	22%	24%	28%

Figure 6. Optimal asset allocation in UK

